education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

SEPTEMBER 2020

PREPARATORY EXAMINATIONS

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 9 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Solve for *x*:

1.1.1 $x(\pi - x) = 0$, where *x* is a rational number. (2)

1.1.2 $-3x^2 + 8x = -7$ (rounded off to 2 decimal places) (4)

1.1.3 $\sqrt{11 - x} - x = 1$ (5)

1.1.4 $2.3^x = 57 - 3^{x-2}$ (4)

1.1.5 $4x^2 - 5x \leq 0$ (4)

1.2 Determine the values of *x* and *y* if (5)

$$2x + y = 3 \quad and \quad y^2 = x^2 + y + x$$

[24]

QUESTION 2

The first three terms of the first differences of a quadratic sequence are 102 ; 108 ; 114 ; …

2.1 Determine between which two consecutive terms is the first difference 2022. (3)

2.2 Determine the n^{th} term of the sequence if it is further given that the third term of the quadratic sequence is equal to 310. (4)

[7]

QUESTION 3

3.1 The first four terms of an arithmetic sequence are: $\frac{2}{5}$; $\frac{3}{5}$; 0,8 ; 1 ; ...

3.1.1 Determine the value of the n^{th} term. (2)

3.1.2 Calculate the sum of the first 30 terms. (3)

3.2 Given: 2 + 5 + 8 + … to n terms = 72710. Calculate the number of terms in the series. (5)

[10]

QUESTION 4

The sum of the first two terms of a geometric series with positive terms, $r \neq -1$, is four times the sum of the next two terms.
The sum to infinity of this series is 3.

4.1 Show that $r = \frac{1}{2}$. (4)

4.2 Calculate the value of the first term. (2)

[6]

QUESTION 5

Given $g(x) = \frac{1}{2(x+3)} - 1$

5.1 Write down the equations of the vertical and horizontal asymptotes of g. (2)

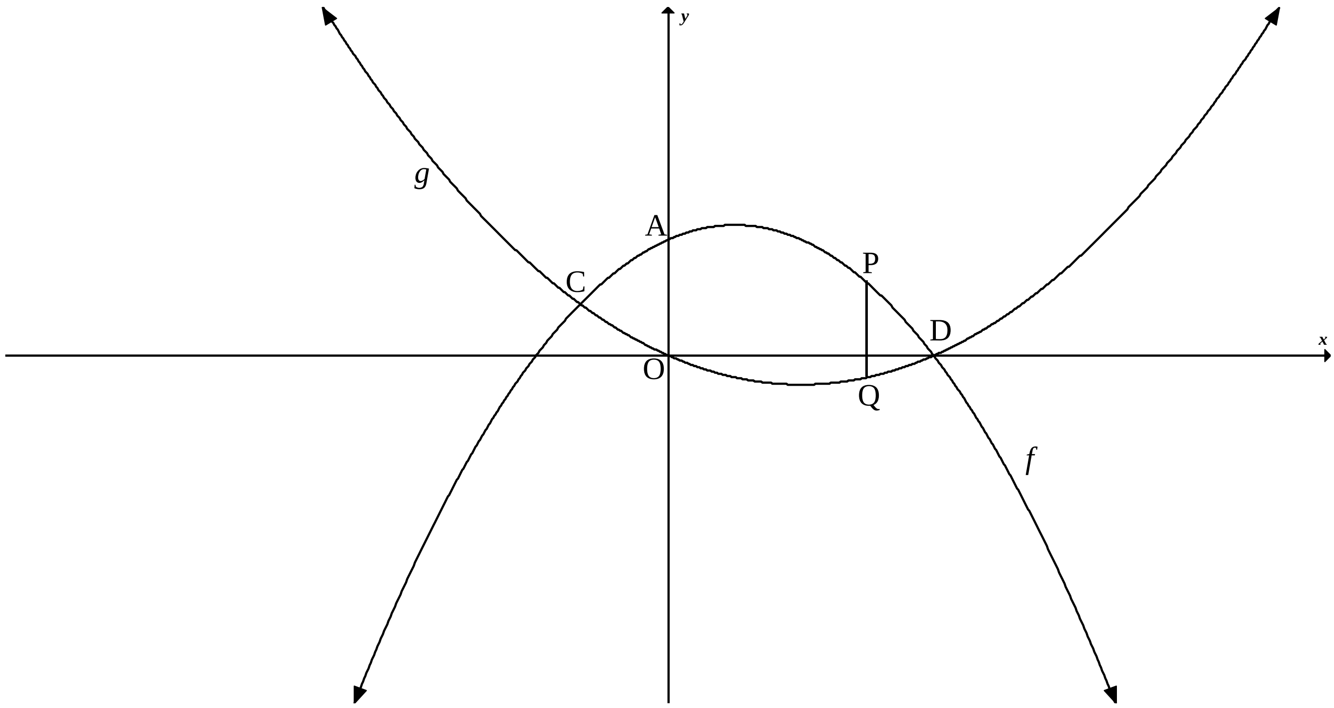
5.2 Determine the intercepts of the graph of g with the axes. (3)

5.3 Draw the graph of g. Show all intercepts with the axes as well as the asymptotes of the graph. (4)

[9]

QUESTION 6

In the diagram, the graphs of $f(x) = -x^2 + x + 2$ and $g(x) = \frac{1}{2}x^2 - x$ are drawn below. f and g intersect at C and D. A is the y – intercept of f. P and Q are any points on f and g respectively. PQ is parallel to the y – axis.

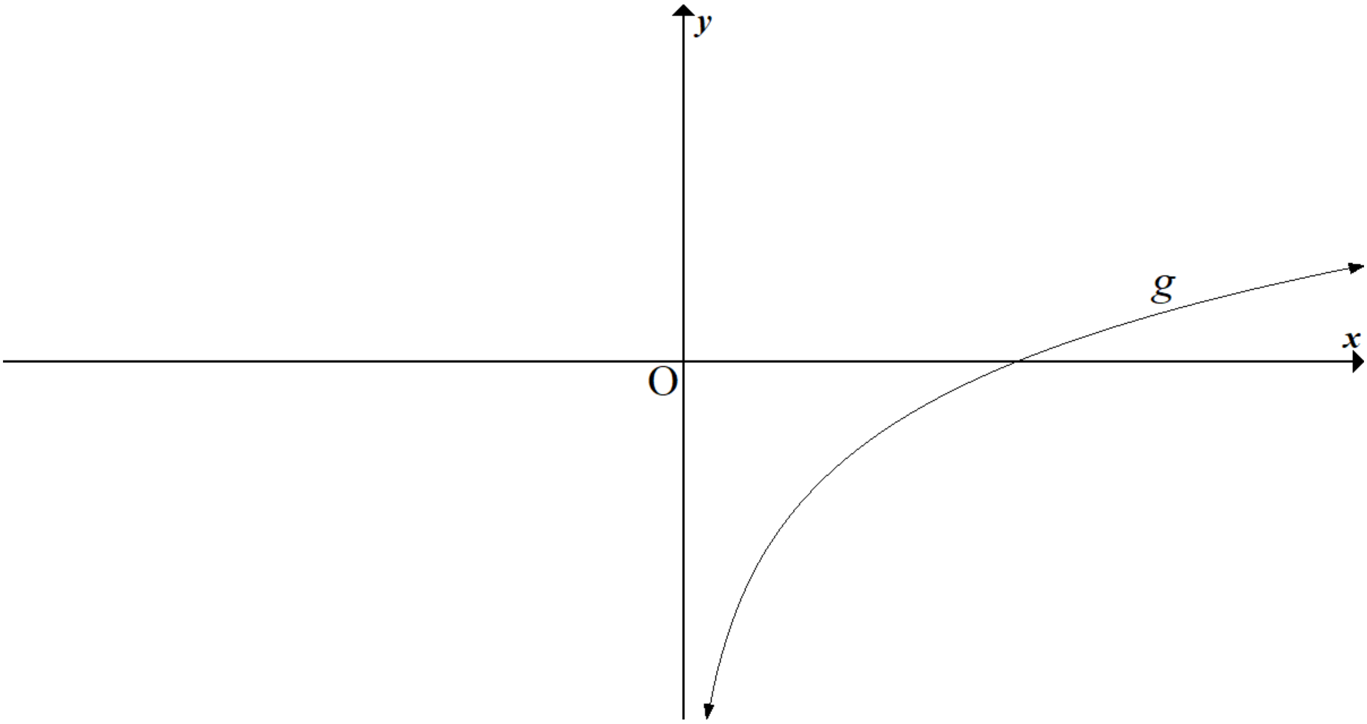


6.1	Write down the co-ordinates of A.	(1)
6.2	Calculate the coordinates of C and D.	(5)
6.3	Determine the values of x for which $f(x) \leq g(x)$.	(2)
6.4	Calculate the maximum length of PQ where line PQ is between C and D.	(4)
6.5	Calculate the value of x where the gradient of f is equal to 3.	(3)
6.6	Determine the values of k for which $f(x) = k$ has two positive unequal roots.	(4)

[19]

QUESTION 7

In the diagram, the graph of $g(x) = log_3 x$ is drawn.



7.1 Write down the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (2)

7.2 Write down the range of g^{-1}. (1)

7.3 Calculate the values of x for which $2g(x) \leq -6$. (4)
[7]

QUESTION 8

8.1 An investor indicates that he will be able to treble the value of the investment at the end of 6 years. The interest rate is fixed and compounded monthly. Calculate the annual interest rate that the investor has on offer. (4)

8.2 Samuel decided to buy a car costing R192 000. He takes out a loan for 5 years at an interest rate charged at 12 % p.a. compounded monthly. Payments are made at the end of each month.

8.2.1 Calculate the monthly repayments over a period of 5 years. (4)

8.2.2 After Samuel had made 45 payments, he decides to settle the balance on the loan. Calculate the lump sum that he will need to pay off the loan after he has made the 45^{th} payment. (4)

[12]

QUESTION 9

9.1 Determine $f^{/}(x)$ from first principles given $f(x) = x^2 - bx$, where b is a constant. (5)

9.2 Determine:

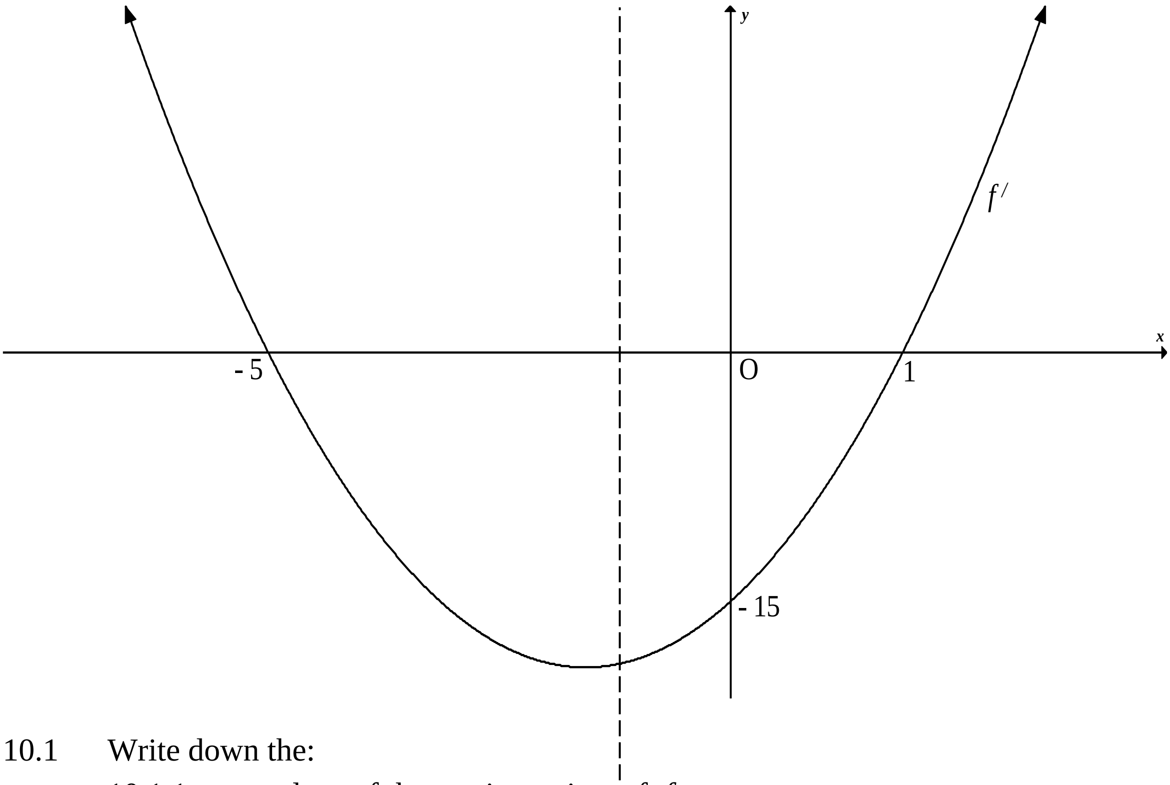
9.2.1 $\dfrac{d}{dx}\left[\dfrac{x^4}{4} - 3.\sqrt[3]{x} + 7\right]$ (3)

9.2.2 $\frac{dy}{dx}$ if $y = (x^{\frac{1}{3}} - 2x^{\frac{2}{3}})^2$ (4)

[12]

QUESTION 10

The graph of $f^{/}$, the derivative of *f*, is drawn below. $f(x) = ax^3 + bx^2 + cx + d\,;a \neq 0$.
$f^{/}$ intersects the *x* – axis at – 5 and 1 and the *y* – axis at – 15.



10.1 Write down the:

10.1.1 *x* – values of the turning points of *f*. (2)

10.1.2 *x* – value(s) where the gradient of *f* is equal to – 15. (2)

10.2 Show that the equation of $f^{/}$ is given by $y = 3x^2 + 12x - 15$. (3)

10.3 If $f(-3) = 0$, calculate the value of *d*. (4)

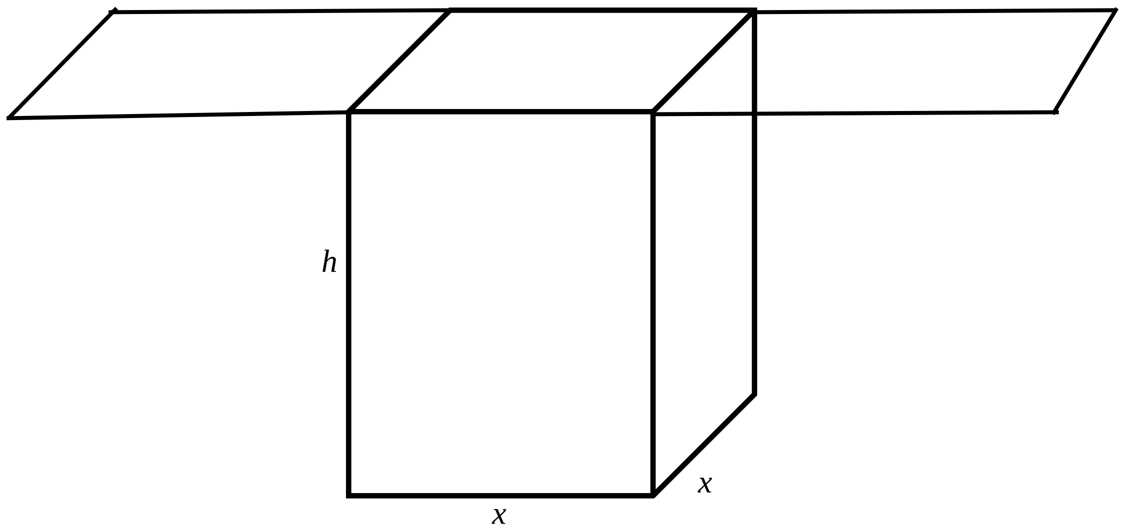
10.4 Determine the coordinates of the turning points of the graph of *f* and state whether they are maximum or minimum turning points. (4)

10.5 $y = tx + 4$ is a tangent to *f*. Calculate the value of *t*. (5)

[20]

QUESTION 11

The rectangular milk carton has a square base which holds 1 litre of milk. It has a specially designed fold-in top. The area of the cardboard used for the top is three times the area of the base.



11.1 Show that the Total Surface Area of the carton is given by (3)

$$A(x) = 4x^2 + \frac{4000}{x}$$

11.2 Determine the dimensions of the carton so that minimum amount of cardboard is used. (6)

[9]

QUESTION 12

A printing company uses 3 machines, A,B and C, to produce banners.

- Machine A produces 20 % of the total production.
- Machine B produces 30 % of the total production.
- Machine C produces 50 % of the total production.
- 2 % of Machine A copies are not perfect.
- 3 % of Machine B copies are not perfect.
- 8 % of Machine C copies are not perfect.

Let P = perfect and NP = not perfect.

12.1 Represent the information by means of a tree diagram. (4)

12.2 A banner is selected at random from the total production. Calculate the probability:

12.2.1 that the banner selected at random was produced by Machine B and is not perfect. (1)

12.2.2 that the banner selected at random is not perfect. (3)

[8]

QUESTION 13

In a survey conducted by 220 grade 12 learners in a school, the following data were collected.

	Like ice-cream (L)	Do not like ice-cream(D)	Total
Boys (B)	65	30	
Girls (G)	70	55	
Total			

13.1 Determine the percentage of boys that like ice-cream. (2)

13.2 Calculate the probability that a randomly selected boy likes ice-cream. (2)

13.3 Are the events of being a boy and liking ice-cream independent or not? Show working. (3)

[7]

Total: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum f.x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$